Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel Level 3 GCE

Paper reference **8FM0/22**

Further Mathematics

Advanced Subsidiary
Further Mathematics options
22: Further Pure Mathematics 2
(Part of option A only)

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear.
- Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 40. There are 5 questions.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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Q:1/1/1/1/

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Pearson

1. Sketch on an Argand diagram the region defined by

$$\left\{z \in \mathbb{C}: -\frac{\pi}{4} < \arg(z+2) < \frac{\pi}{4}\right\} \cap \{z \in \mathbb{C}: -1 < \mathrm{Re}(z) \leqslant 1\}$$

On your sketch

- shade the part of the diagram that is included in the region
- use solid lines to show the parts of the boundary that are included in the region
- use dashed lines to show the parts of the boundary that are not included in the region

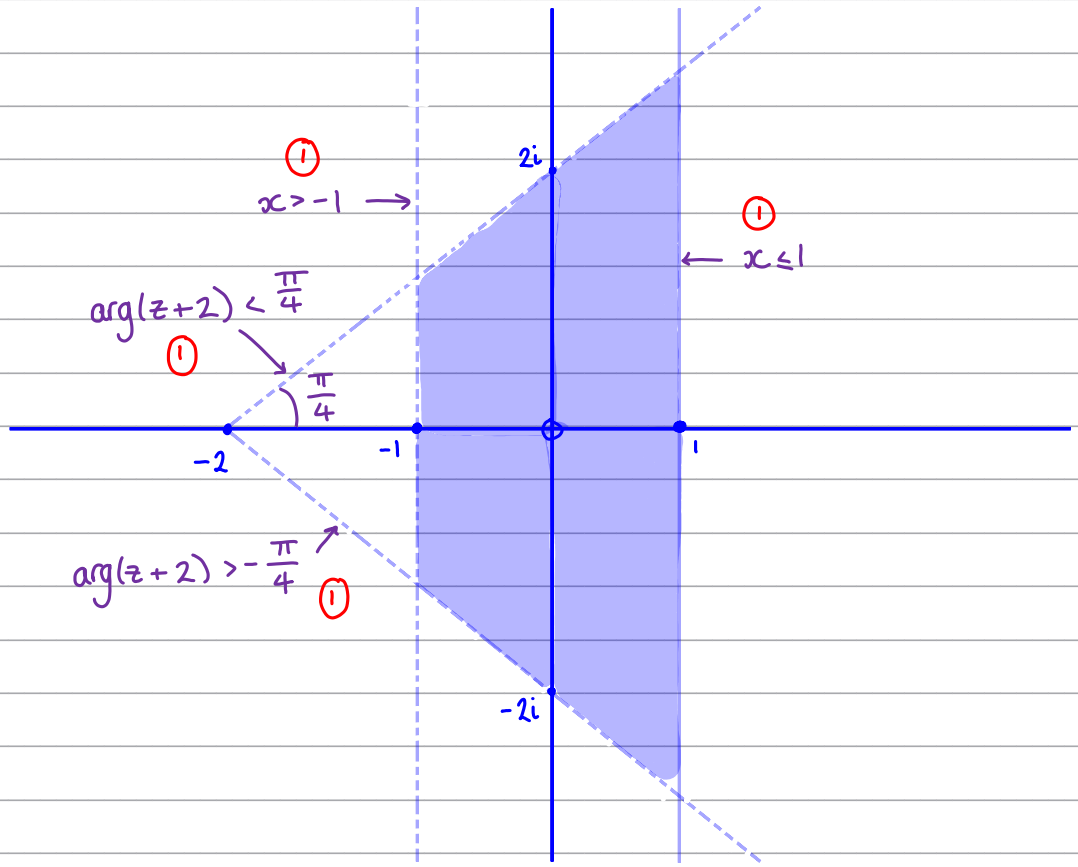
(4)

$\arg(z-z_1) = \theta$ is a half-line from z_1 at angle θ to the horizontal, giving us two boundaries:

$\arg(z+2) = \frac{\pi}{4}$ $\arg(z+2) = -\frac{\pi}{4}$

The real part of z is bound by $-1 < \mathrm{Re}(z) \leq 1$, so this gives us two more boundaries

$x = -1$ $x = 1$



* purple labels not required - these marks are for each part of the correct diagram.

Question 1 continued

(Total for Question 1 is 4 marks)

2. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

$$\mathbf{M} = \begin{pmatrix} 4 & 2 \\ 3 & -1 \end{pmatrix}$$

Find a matrix **P** and a diagonal matrix **D** such that

$$\mathbf{P}^{-1}\mathbf{MP} = \mathbf{D}$$

(7)

$\det(M - \lambda I) = \left| \begin{pmatrix} 4 & 2 \\ 3 & -1 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right|$ ← find characteristic equation.

$0 = \begin{vmatrix} 4-\lambda & 2 \\ 3 & -1-\lambda \end{vmatrix}$ ← solve to get eigenvalues

$0 = (4-\lambda)(-1-\lambda) - 2\times 3$ (1) ← $\det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc$

$0 = -4 + \lambda - 4\lambda + \lambda^2 - 6$

$0 = \lambda^2 - 3\lambda - 10$

$0 = (\lambda - 5)(\lambda + 2)$

∴ Eigenvalues are $\lambda = 5$, $\lambda = -2$. (1)

When $\lambda_1 = -2$: ← find eigenvector v_1

$Mv_1 = \lambda_1 v_1 \Rightarrow \begin{pmatrix} 4 & 2 \\ 3 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = -2 \begin{pmatrix} x \\ y \end{pmatrix}$

$4x + 2y = -2x$

$3x - y = -2y$

$3x = -y$ (1) $\Rightarrow v_1 = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$ (1)

use $x = 1$, then $y = -3 \times 1 = -3$

Question 2 continued

when $\lambda_2 = 5$: ← find eigenvector v_2

$$Mv_2 = \lambda_2 v_2 \Rightarrow \begin{pmatrix} 4 & 2 \\ 3 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = 5\begin{pmatrix} x \\ y \end{pmatrix}$$

$$\left.\begin{aligned} 4x + 2y &= 5x \\ 3x - y &= 5y \end{aligned}\right\} \quad 2y = x \Rightarrow v_2 = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \quad (1)$$

Hence, $P = \begin{pmatrix} 1 & 2 \\ -3 & 1 \end{pmatrix}$ (1) (v_1, v_2) and $D = \begin{pmatrix} -2 & 0 \\ 0 & 5 \end{pmatrix}$ (1) (λ_1, λ_2)

Question 2 continued

Question 2 continued

(Total for Question 2 is 7 marks)

3. (i) Let G be a group of order 5 291 848

Without performing any division, use proof by contradiction to show that G cannot have a subgroup of order 11

(3)

(ii)(a) Complete the following Cayley table for the set $X = \{2,4,8,14,16,22,26,28\}$ with the operation of multiplication modulo 30 ← if $x \times y > 30$, subtract 30.

$\times_{30}$	2	4	8	14	16	22	26	28
2	4	8	16	28	2	14	22	26
4	8	16	2	26	4	28	14	22
8	16	2	4	22	8	26	28	14
14	28	26	22	16	14	8	4	2
16	2	4	8	14	16	22	26	28
22	14	28	26	8	22	4	2	16
26	22	14	28	4	26	2	16	8
28	26	22	14	2	28	16	8	4

(1) 1 row

(1) 5 rows

(1) all rows

A copy of this table is given on page 11 if you need to rewrite your Cayley table.

(b) Hence determine whether the set X with the operation of multiplication modulo 30 forms a group.

[You may assume multiplication modulo n is an associative operation.]

(6)

(a) If G has a subgroup of order 11, then by Lagrange's Theorem, 5291848 must be divisible by 11 (1)

$5 - 2 + 9 - 1 + 8 - 4 + 8 = 23$ (1) ← using divisibility test for 11.

23 is not divisible by 11, therefore 5291848 is not divisible by 11, which contradicts Lagrange's Theorem. Hence, there is no subgroup of order 11. (1)

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Question 3 continued

(c) The row and column of the element 16 repeat the ordering of the table borders, so 16 is an identity element of $(X, \times_{30})$. (1)

identity.
← $a * e = e$

Each element has an inverse as follows: (1)

inverse:
← $a * b = b * a = e$
(find 16 in the table, the row and column are inverse)

x	2	4	8	14	16	22	26	28
x^{-1}	8	4	2	14	16	28	26	22

$\times_{30}$ is associative

All elements in the table are elements of the set X, therefore $(X, \times_{30})$ is closed (1)

$(X, \times_{30})$ satisfies the group axioms, so $(X, \times_{30})$ is a group.

Question 3 continued

Question 3 continued

Only use this grid if you need to rewrite your Cayley table.

$\times_{30}$	2	4	8	14	16	22	26	28
2	4	8	16	28	2	14	22	26
4	8		2			28	14	
8	16	2			8			14
14	28		22	16		8	4	
16	2	4		14	16			
22	14		26			4	2	16
26	22	14		4				8
28	26		14		28		8	

(Total for Question 3 is 9 marks)

4. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

(i) (a) Use the Euclidean algorithm to find the highest common factor h of 416 and 72

(3)

(b) Hence determine integers a and b such that

$$416a + 72b = h$$

(3)

(c) Determine the value c in the set $\{0,1,2...,415\}$ such that

$$23 \times 72 \equiv c \pmod{416}$$

(2)

(ii) Evaluate $5^{10} \pmod{13}$ giving your answer as the smallest positive integer solution.

(3)

(i) (a) $416 = 5 \times 72 + 56$ (1) ← Set $a = 416$ and $b = 72$ such that $a \geq b$.

$72 = 1 \times 56 + 16$ ← Apply $a = q_1 b + r_1$ then $b = q_2 r_1 + r_2$ ... until $r_2 = 0$

$56 = 3 \times 16 + 8$

$16 = 2 \times 8 + 0$ (1) ← Then $\gcd(a,b) = r_1$

(r_1 = 8, r_2 = 0)

$\therefore h = 8$ (1)

(b) $8 = 56 - 3 \times 16$ (1) ← Using back-substitution.

$8 = 56 - 3(72 - 1 \times 56)$ ← from (a), $72 = 56 + 16$, $16 = 72 - 56$

$8 = 4 \times 56 - 3 \times 72$

$8 = 4 \times (416 - 5 \times 72) - 3 \times 72$ (1) ← from (a), $416 = 5 \times 72 + 56$, $56 = 416 - 5 \times 72$

$8 = 4 \times 416 - 23 \times 72$ ← $416a + 72b = h$

$\therefore a = 4$, $b = -23$ (1)

Question 4 continued

(c) $23 \times 72 \equiv 4 \times 416 - 8$ ← rearrange (b)

$\equiv -8 \pmod{416}$ ①

$\therefore c = 416 - 8 = 408$ ①

(ii) $5^{10} \equiv (5^2)^5 \equiv 25^5 \equiv (-1)^5$ ① $\pmod{13}$

↑ $-1 \bmod 13 \equiv 25$ $(25 - 2\times13 = -1)$

$(-1)^5 = -1 \Rightarrow 5^{10} \equiv -1 \bmod 13$ ①

$-1 + 13 = 12 \Rightarrow 12 \bmod 13$

$\therefore 5^{10} \equiv 12 \bmod 13$ ①

Question 4 continued

Question 4 continued

(Total for Question 4 is 11 marks)

5. A person takes a course of a particular vitamin.

Before the course there was none of the vitamin in the person's body.

During the course, vitamin tablets are taken at the same time each day.

Initially two tablets are taken and on each following day only one tablet is taken.

Each tablet contains 10 mg of the vitamin.

Between doses the amount of the vitamin in the person's body decreases naturally by 60%

Let u_n mg be the amount of the vitamin in the person's body immediately after a tablet is taken, n days after the initial two tablets were taken.

(a) Explain why u_n satisfies the recurrence relation

$$u_0 = 20 \qquad u_{n+1} = 0.4u_n + 10$$

(2)

The general solution to this recurrence relation has the form $u_n = a(0.4)^n + b$

(b) Determine the value of a and the value of b.

(4)

The course is only effective if the amount of the vitamin in the person's body remains above 6 mg at **all times** throughout the course.

(c) Determine whether this course of the vitamin will be effective for this person, giving a reason for your answer.

(3)

(a) Immediately after the first dose there is $2 \times 10\text{mg} = 20\text{mg}$ in the body, so $u_0 = 20$.

Reduction by 60% daily means that right before the next dose, $0.4u_n$ mg is retained in the body. ① 2/3 statements

The next tablet adds 10mg, giving $u_{n+1} = 0.4u_n + 10$. ① 3/3 statements

Question 5 continued

(b) at u_0: $20 = a + b$ ①

at u_1: $u_1 = 0.4(20) + 10 \Rightarrow u_1 = 18$ ①

$18 = 0.4a + b$ ②

① − ②: $20 - 18 = a + b - (0.4a + b)$ ①

$2 = a(1 - 0.4)$

$2 = 0.6a$

$\frac{10}{3} = a$ ①

$20 = \frac{10}{3} + b$

$b = 20 - \frac{10}{3}$

$b = \frac{50}{3}$ ①

(c) $u_n = \frac{10}{3}(0.4)^n + \frac{50}{3}$

Consider the 'long term' as n increased, or n tends toward (→) ∞.

As $n \to \infty$, $(0.4)^n \to 0$ ∴ $u_n \to \frac{50}{3}$ ①

Minimum amount in body occurs just before the next 10 mg tablet is taken, so:

$\min(u_n) = \frac{50}{3} - 10 = 6\frac{2}{3}$ mg ①

∴ There will always be at least 6 mg of the vitamin in the body, so the course of the vitamin will be effective ①

Question 5 continued

Question 5 continued

Question 5 continued

(Total for Question 5 is 9 marks)

TOTAL FOR FURTHER PURE MATHEMATICS 2 IS 40 MARKS